An Efficient Direct Search Methodology for Robust Optimizations of Electromagnetic Devices

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To eliminate the deficiency in the biasing force selection and mathematical formulation of existing robust optimizers, a direct search formulation is proposed to treat the robust performances as additional constrained functions. To solve the resulting robust optimization problem, a robust oriented quantum-inspired evolutionary algorithm (QEA) is proposed. In the proposed robust QEA, a mechanism to activate the robust performance checking procedure only to promising solution is introduced, and efficiently numerically implemented. The numerical results on a case study are reported in order to validate the feasibility and the merit of the proposed methodology in solving practical engineering robust optimization designs.

***Index Terms*— Quantum-inspired evolutionary algorithm, robust optimization, uncertainty.**

I. A Direct Search Methodology for Robust Optimizations

In the study of design optimizations of electromagnetic devices, increasing attentions have been given to the robustness of an optimal design since it is possible that slight perturbations or variations in the optimized variables obtained using a traditional performance based optimizer could result in either a significant performance degradation or an infeasible solution due to the violation of the design constraint functions because of the existences of inevitable and unavoidable imprecision and uncertainties in an engineering design problem. Consequently, a wealth of robust oriented optimization techniques have been developed and applied successfully to solve different engineering design problems under conditions of uncertainties in electromagnetics [1]-[3].

Robustness means some degree of insensitivity to small perturbations in either the design or environmental variables. To quantify the uncertainty in robust design optimizations, there are two categories of uncertainty quantization, the probability-based and the interval-based approaches [4]. The probability-based approach uses some probabilistic information of the uncertainty, commonly the mean (expected fitness) and the standard deviation as the gauges to assess the robustness of a solution. The interval-based method simply uses the nominal value and the bounds of the uncertain parameters, and the worst case scenario is commonly used as the robust performance.

Obviously, an ensemble of tens or even hundreds of function calls in a small neighborhood of a given solution is sampled and their function values are used to quantify the uncertainty in robust optimizations. In other words, the computational burden for a robust optimizer is significantly higher than that for its global counterpart. This situation is further exacerbated by the application of high fidelity numerical methods, such as finite element models, which are commonly used in inverse and optimization problems. In this regard, an efficient direct search methodology is proposed in this paper.

A. New Formulation of a Robust Optimization

Without loss of generality, one considers a constrained minimization problem with an interval uncertainty, as formulated as:

$$\begin{aligned} &\text{Min} \quad f(x+\delta) \\ &\text{S.T.} \quad g_i(x+\delta) \le 0 \quad (i=1,2,\ldots,k) \\ &\quad \delta_N-\delta_0 \le \delta \le \delta_N+\delta_0 \end{aligned} \tag{1}$$

where, x is the vector of design (decision) parameters (variables), δ is the vector of uncertainty variables, δ_N is the vector of the nominal value of δ, δ_0 is the vector of the half range of the interval uncertainty.

In this paper, the standard deviation and the worst case metric are used as the robust performance parameters, respectively, for the objective and constraint functions, i.e.:

$$\sigma[f(x+\delta)]=\sqrt{\frac{1}{N_s}\sum_{1}^{N_s}\{f(x_i+\delta_N)-\bar{f}(x_i+\delta_N)\}^2} \tag{2}$$

$$(g_i)_w(x)=\underset{(\delta_N-\delta_0\le\delta\le\delta_N+\delta_0)}{Max}[g_i(x+\delta)] \tag{3}$$

where N_s is the number of total sampling points generated in a small neighborhood of the specific point x.

In most of the robust optimization methodologies , the robust counterparts of the original design problem of (1), such as formulated in (2) and (3), are either simply solved in lieu of the original optimal design problem, or combined with the original performance parameter (objective function and constraints) to transfer to a multi-objective optimization problem to seek the robust optimal solution. In other words, the robust performance is used as a predominant parameter in determining the searching direction of an optimizer in the optimization process. However, it is mathematically known that only local optimal solutions and boundary solutions have the potential to be a robust optimal solution [5]. In this regard, the original objective function should be selected as the biasing force of the optimizer to stimulate enough competitive pressure to evolve to the exact robust optimal solution. Consequently, the proposed robust optimization formulation is directly formulated as

$$\begin{aligned} &\text{Min} \quad f(x+\delta_N) \\ &\text{Subject to} \quad g_i(x+\delta_N) \le 0 \quad (i=1,2,\ldots,k) \end{aligned}, \tag{7}$$

together with,

$$\begin{cases}(g_i)_w(x) - g_i(x+\delta_N) \le (\Delta g_i)_{tolerance} \\ \sigma[f(x+\delta)] \le \Delta_{tolerance} \\ (\delta_N - \delta_0 \le \delta \le \delta_N + \delta_0)\end{cases} \quad (8)$$

where $g_w(x)$ is the worst case scenario of $g(x)$, $(\Delta g)_{tolerance}$ is the acceptable tolerance of $g(x)$, $\sigma[f(x)]$ is the standard deviation of $f(x)$, $\Delta_{tolerance}$ is the acceptable tolerance of $\sigma[f(x)]$.

B. A Robust Oriented Quantum-Inspired Evolutionary Algorithm

Even though any evolutionary algorithm can be used readily to find the robust optimal solution of (7) and (8), the quantum-inspired evolutionary algorithm (QEA) [6] is extended to a robust oriented optimizer due to its inherent characteristics in developing an efficient robust optimizer.

Mathematically, the robust optimal solution of a constrained optimal design problem is either a local/global optimum of the objective function or a solution distributed on the boundaries of the feasible parameter space. In this point of view, it is unnecessary to check the robust performance constraints of (8) for every intermediate individual which is generated by observing the *Q*-bit individual. However, it is not easy to identify if an intermediate solution is a (local) optimal one in the optimization process. Nevertheless, from the iterative procedures of the proposed QEA, it is obvious that only the best and boundary solutions ever searched have the potential to be a global/local optimum. This salient feature lends that the proposed algorithm is ideal for developing an efficient and simple robust optimizer since one needs only to check the robust performance constraints of those potential "optimal" solutions. In this point of view, a simple strategy to activate the robust performance checking procedure is proposed. More specially, in the iterative process, the robust performance feasibility checking of (8) is activated only when a new best solution/a boundary solution, x^*, is searched/generated and iff $\|\nabla f(x^*)\|_2$ is smaller than a predefined value. Obviously, the robust performance feasibility checking of the mere potential solutions rather than the total intermediate ones will reduce a huge amount of computation costs. Moreover, to efficiently determine $\|\nabla f(x^*)\|_2$ the stochastic approximation method is employed to realize computational savings of *n* (*n* is the number of the decision variables) times relative to the finite difference approximation [7].

II. Numerical Validation and Conclusions

The robust optimization counterpart of the Team Workshop problem 22 of a superconducting magnetic energy storage (SMES) configuration with three free parameters [8] is selected and solved to validate the feasibility and merit of the proposed methodology. For performance comparisons, this case study is solved, respectively, by using the proposed robust optimal methodology, the combined Polynomial Chaos and PSO approach (PCPSO) [8]. As explained in [8], the combined PCPSO approach is proposed for efficiently solving robust optimization under uncertainties with special random process. In the numerical implementation of the combined PCPSO approach, a Gaussian random process is assumed for the uncertainties in the design variables. Moreover, to facilitate implementations of polynomial chaos, the distance (radial) variable in the feasible space is used as the only uncertain variable. For a fair comparison, the final robust optimal solution obtained by using the combined PCPSO is used as a reference.

In the numerical study, the interval uncertainty is set to ± 1% limits of the ranges of the corresponding decision variables. The final solutions searched by using the proposed and the PCPSO method are nearly identical, while the iterative numbers for the proposed and the PCPSO method in a typical run are, respectively, 2685 and 3018. More specially, for the robust optimal solution, the stray field is 7.549×10^{-7}, the stored energy is 179.2140 MJ; while these are, respectively, to 7.75×10^{-7}, 179.9956 MJ for the global optimal solution which is obtained using a traditional performance-based optimizer.

Moreover, to evaluate the robustness of the global and robust optimal solutions against small variations, some post-processing numerical experiments are conducted. In a more detail description, 10 random perturbations with ± 1% limits of the bound of the interval uncertainty are applied to the two optimized decision variables. The numerical results have revealed that the averaged performance degradations of the global optimal solution for the 10 perturbations are that the stray field changing is 0.82 and the averaged deviation of the stored energy is 0.02, both in relative values; while that the averaged performance degradations of the robust optimal solution obtained using the two robust optimizers for the 10 perturbations are that the stray field changing is 0.33, less than the given tolerance of 35%, and the averaged deviation of the stored energy is 0.0015, nearly identical to the given tolerance of 0.15%, both in relative values.

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